Warm Up

The US population from 1790 to 1990 can be modeled by $p(x) = 0.0057x^3 + 0.4895x^2 + 0.3236x + 3.8431$, where x is the number of decades after 1790. Use the end behavior of the graph to describe the population trend. Support the conjecture numerically. (table)

(large x)

as $x \to \infty$, $p(x) \to \infty$

As time passes, the population increases.

x	P
20	251.72
25	406.93
30	608

(million?)

Fri. Sept. 9

Objective: SWBAT graph rational functions & identify their key points

Agenda:
- Warm Up
- HW Huddle
- Notes
- Examples
- Practice
- Reflection

HW: Precalculus pg. 138 10-28 even; no domain

Notes: Rational Functions

onal function is any function f(x) that can be represented as:

$$\frac{a(x)}{b(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_0}$$ (two polynomials)

e $b(x) \neq 0$

) = 0, there are vertical asymptotes or holes.

Notes: Vertical Asymptotes

To find holes/vertical asymptotes:

1. Factor the numerator and denominator
2. Cancel
3. Uncanceled factors in the denominator make *asymptotes*; canceled factors make *holes*

Example: $g(x) = \frac{x^2 - 4}{x^2 - 2x - 8} = \frac{\cancel{(x+2)}(x-2)}{(x-4)\cancel{(x+2)}} = \frac{x-2}{x-4}$

hole: $x+2=0 \rightarrow x=-2$

asymptote: $x-4=0$

$x=4$

Examples

Find the vertical asymptotes and holes.

1. $j(x) = \dfrac{x^2 + 10x + 24}{x^2 + x - 12}$ $= \dfrac{(x+6)\cancel{(x+4)}}{\cancel{(x+4)}(x-3)} = \dfrac{x+6}{x-3}$

hole: $x = -4$

asymptote: $x = 3$

2. $c(x) = \dfrac{x^2 - 2x - 3}{x^2 - 4x - 5}$ $= \dfrac{(x-3)\cancel{(x+1)}}{(x-5)\cancel{(x+1)}}$

hole: $x = -1$

asymptote: $x = 5$

Notes: Other Asymptotes

1. Divide the numerator by the denominator.
2. The quotient gives the equation of a horizontal asymptote if it's constant (or zero).
3. If the quotient is linear, there is an oblique asymptote.

can you find any shortcuts?

ex) $g(x) = \dfrac{2x^2 + 5}{x + 1}$

$$\begin{array}{r|rrr} -1 & 2 & 0 & 5 \\ & & -2 & 2 \\ \hline & 2 & -2 & 7 \end{array}$$

$2x - 2 + \dfrac{7}{x+1}$

asymptote:

- $x = -1$
- $y = 2x - 2$

Example

Find all asymptotes & holes: $\frac{x^2 + 3x - 3}{x + 4} = x - 1 + \frac{1}{x+4}$

$$\begin{array}{r|rrr} -4 & 1 & 3 & -3 \\ & & -4 & 4 \\ \hline & 1 & -1 & 1 \end{array}$$

asymptotes:

- $y = x - 1$
- $x = -4$

Notes: X- and Y-Intercepts

same as always:

- x-intercept: where y = 0
- y-intercept: where x = 0

ex) $f(x) = \frac{3x^2 - 3}{x^2 - 9}$

$y = \frac{3(0)^2 - 3}{(0)^2 - 9} = \frac{3}{9} = \frac{1}{3}$

$x^2-9\,(0) = \left(\frac{3x^2 - 3}{\cancel{x^2 - 9}}\right)\cancel{x^2-9} \rightarrow 0 = 3x^2 - 3$

$+3 \qquad +3$

$\frac{3}{3} = \frac{3x^2}{3} \rightarrow \sqrt{1} = \sqrt{x^2} \rightarrow x = 1, -1$

Practice

Precalculus pg. 138: 9-27 odd

Reflection

How do you find non-vertical asymptotes?